

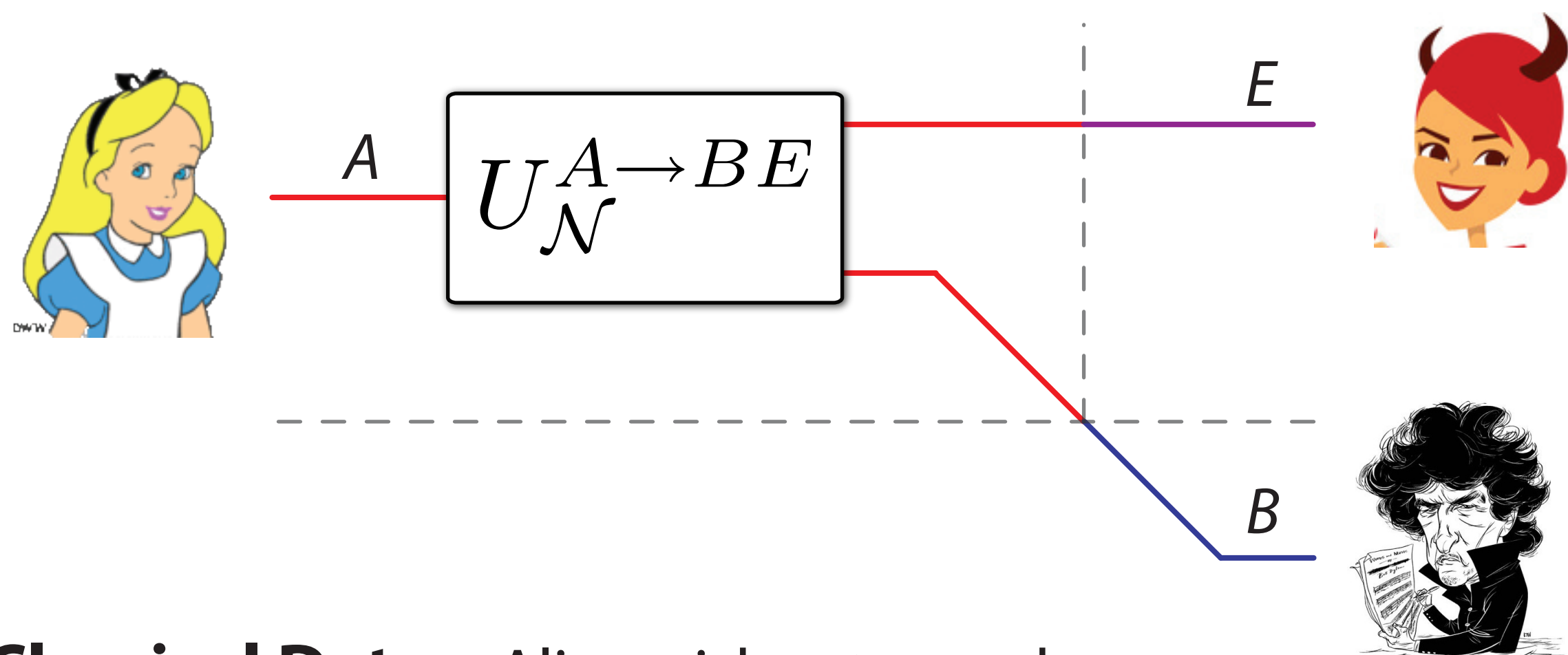
Trading Resources in Quantum Communication

Mark M. Wilde

School of Computer Science, McGill University,
Montreal, Quebec, Canada



Uses of a Quantum Channel



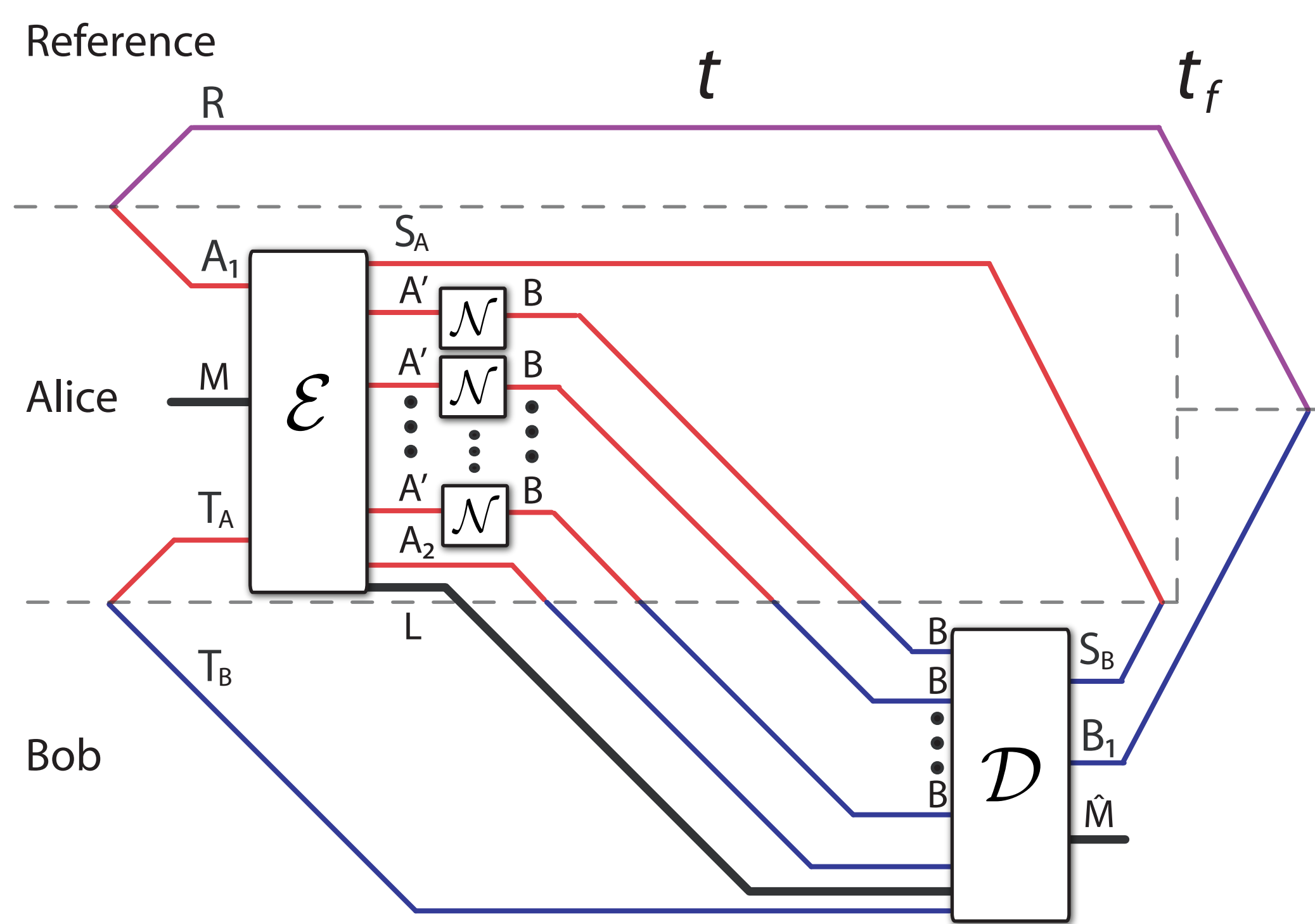
Classical Data: Alice wishes to send "I love you" or "I don't love you"

Quantum Data: Alice wishes to send $\frac{1}{\sqrt{2}}(|\text{"I love you"}\rangle + |\text{"I don't love you"}\rangle)$

Private Classical Data: A concerned Alice sends "I love you" or "I don't love you" and doesn't want Eve to know

Assisting Resources: If Alice and Bob share any assisting resources such as entanglement or secret key, this can help

Trading in the CQE Setting

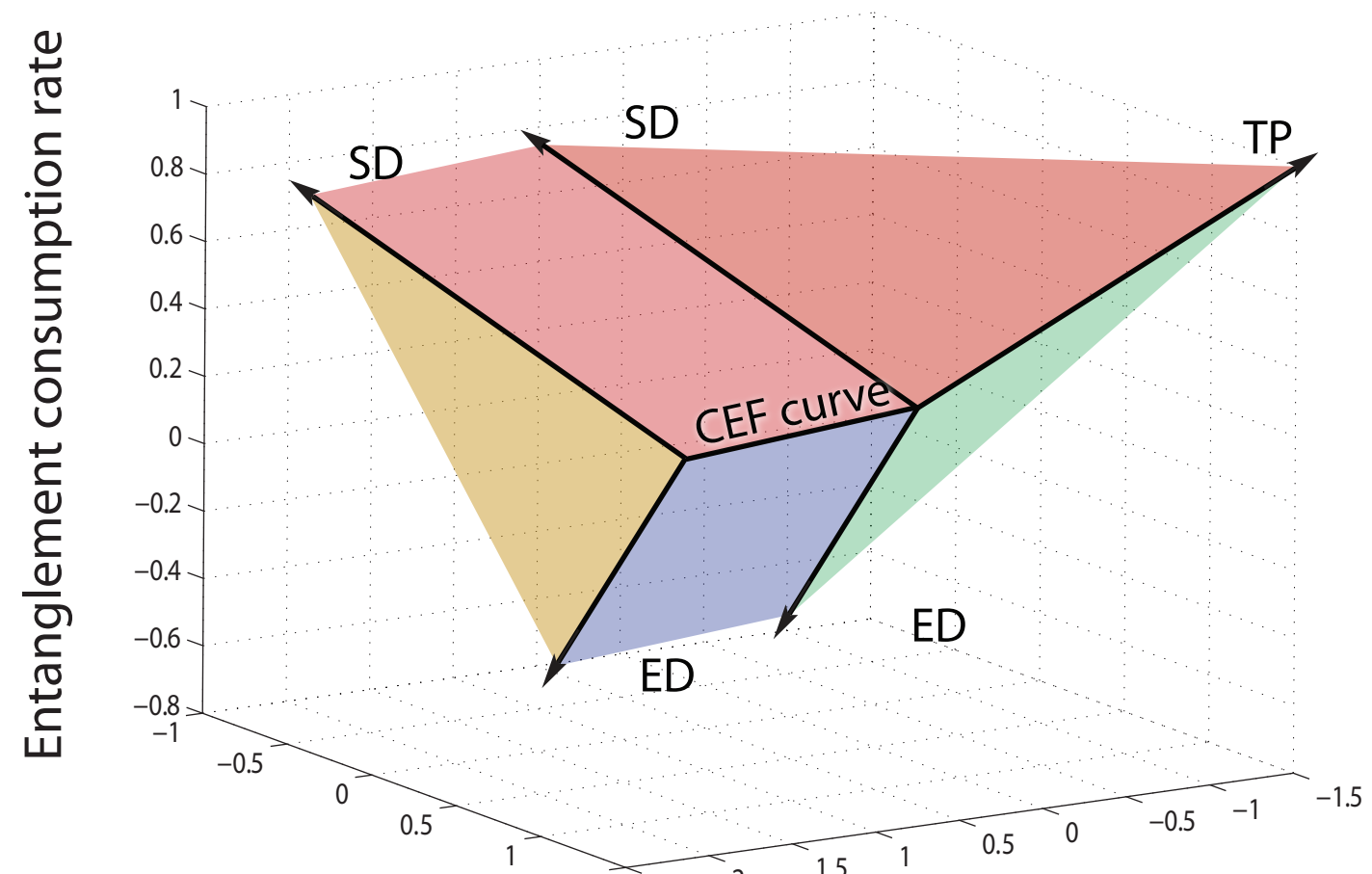
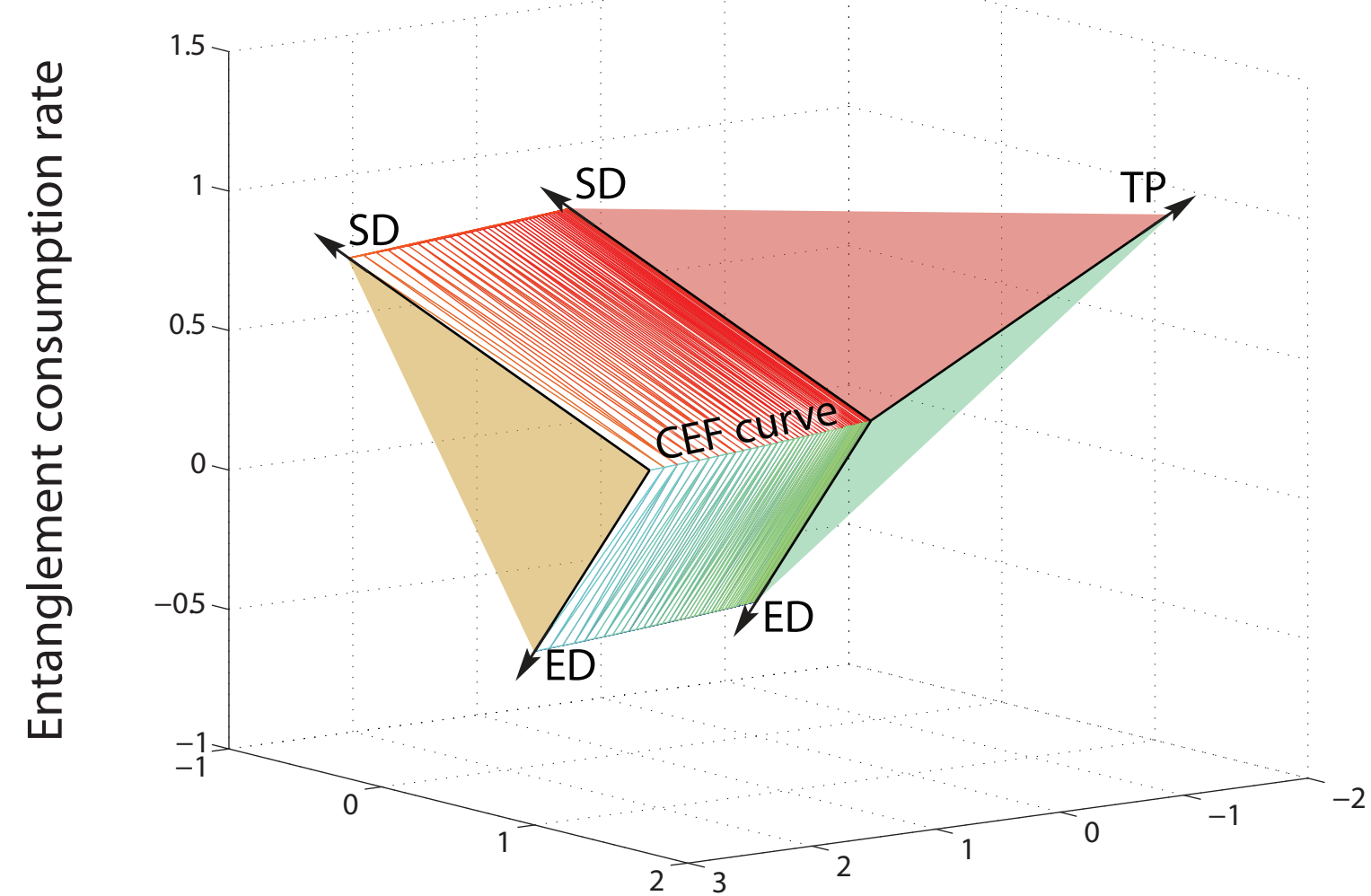


Catalytic protocol: generate and consume classical comm. (C), quantum comm. (Q), and entanglement (E) in addition to channel

Example CQE Capacity Regions

Qubit Dephasing Channel

Qubit Erasure Channel



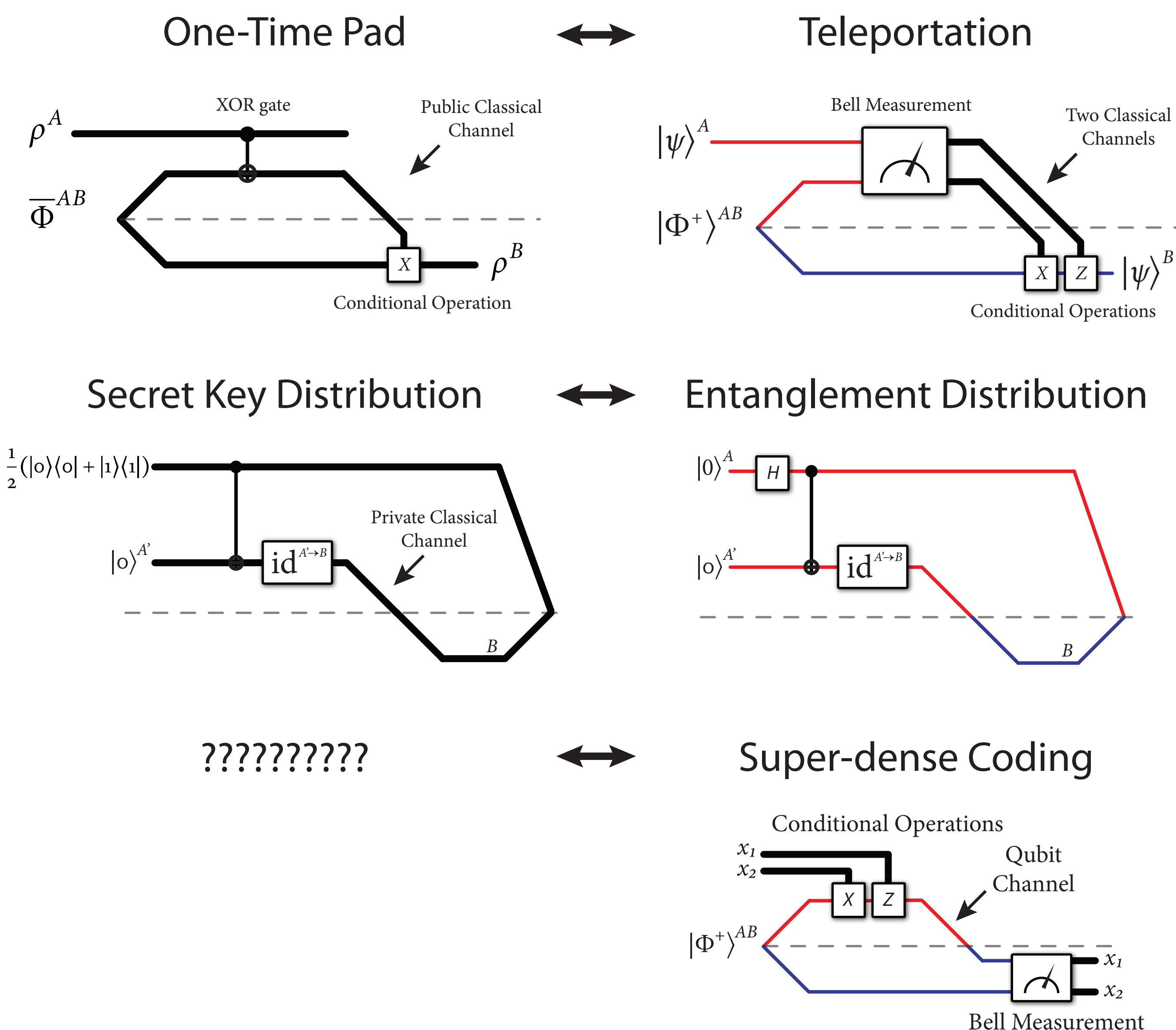
$$\begin{aligned} C + 2Q &\leq 1 + H_2(v) - H_2(\gamma(v, p)), \\ Q + E &\leq H_2(v) - H_2(\gamma(v, p)), \\ C + Q + E &\leq 1 - H_2(\gamma(v, p)) \end{aligned}$$

$$v \in [0, 1/2] \quad \gamma(v, p) \equiv \frac{1}{2} + \frac{1}{2} \sqrt{1 - 16 \cdot \frac{p}{2} \left(1 - \frac{p}{2}\right) v(1-v)}$$

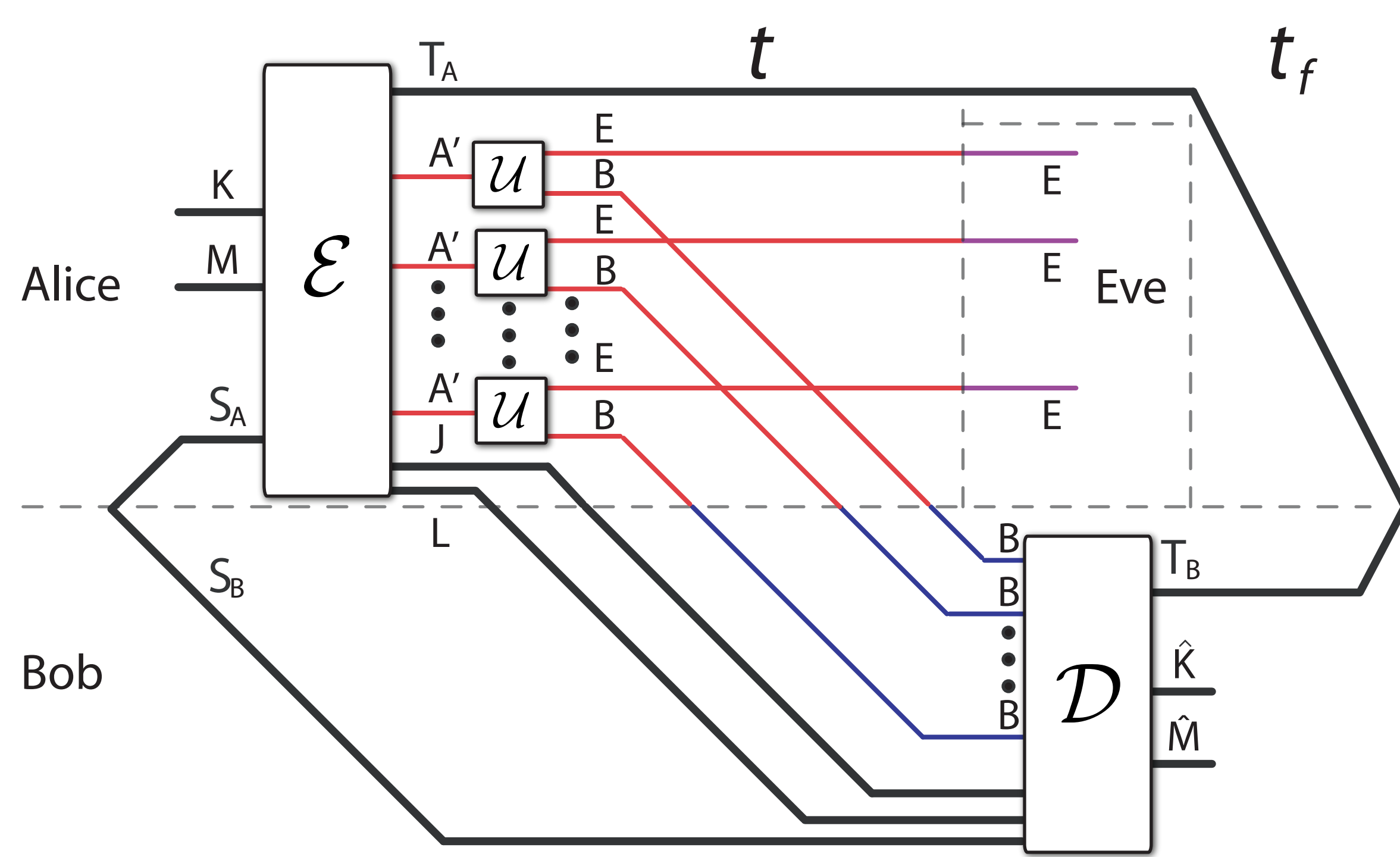
$$\begin{aligned} C + 2Q &\leq (1 - \epsilon)(1 + H_2(p)), \\ Q + E &\leq (1 - 2\epsilon)H_2(p), \\ C + Q + E &\leq 1 - \epsilon - \epsilon H_2(p) \end{aligned}$$

$$p \in [0, 1/2]$$

The Collins-Popescu Analogy



Trading in the RPS Setting

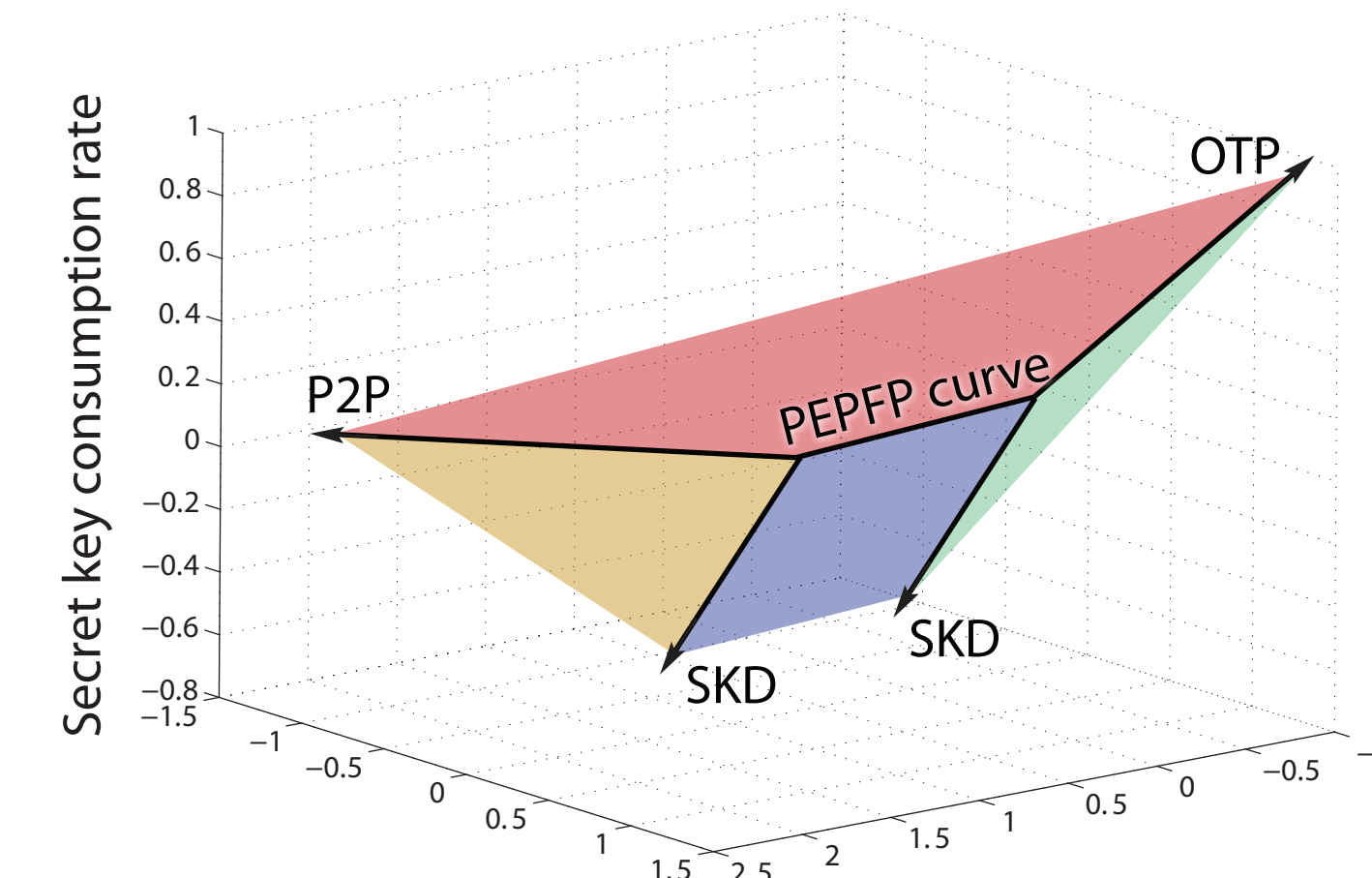
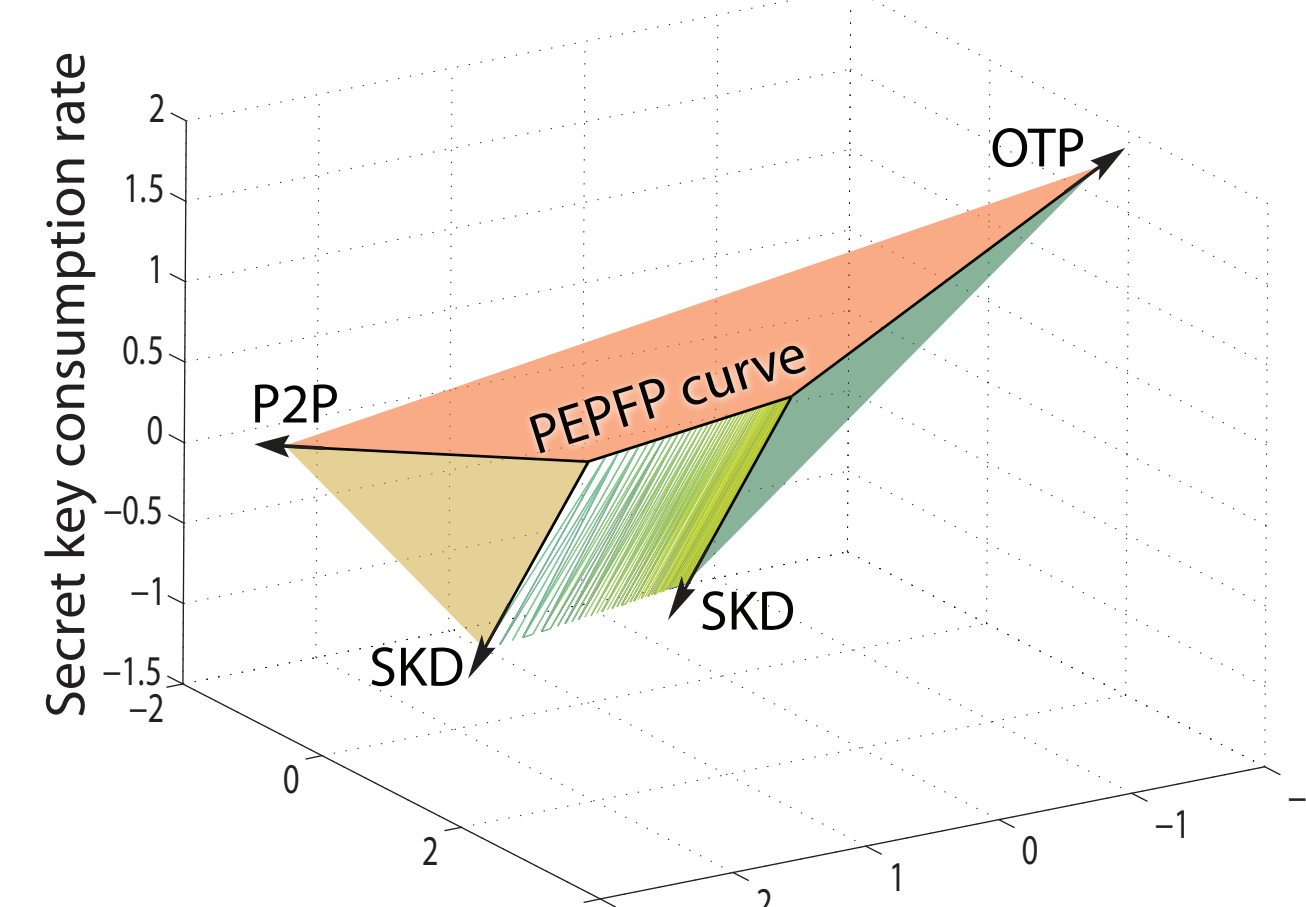


Catalytic protocol: generate and consume public classical comm. (R), private classical comm. (P), and secret key (S) in addition to channel

Example RPS Capacity Regions

Qubit Dephasing Channel

Qubit Erasure Channel



$$\begin{aligned} R + P &\leq 1, \\ P + S &\leq H_2(v) - H_2(\gamma(v, p)), \\ R + P + S &\leq 1 - H_2(\gamma(v, p)) \end{aligned}$$

$$\begin{aligned} R + P &\leq (1 - \epsilon), \\ P + S &\leq (1 - 2\epsilon)H_2(p), \\ R + P + S &\leq 1 - \epsilon - \epsilon H_2(p) \end{aligned}$$

References: arXiv:0811.4227, 0901.3038, 0903.3920, 1004.0458, 1005.3818 (all joint with Min-Hsiu Hsieh)